Introduction to related rates

Related Rates are an application of implicit differentiation, and are usually easy to spot. They ask you to find how quickly one variable is changing when you know how quickly another variable is changing. To solve a related rates problem, complete the following steps:

1. Construct an equation containing all the relevant variables.
2. Differentiate the entire equation with respect to (time), before plugging in any of the values you know.
3. Plug in all the values you know, leaving only the one you're solving for.
4. Solve for your unknown variable.

Example

How fast is the radius of a spherical balloon increasing when the radius is 100 cm, if air is being pumped into it at 400 cm^3/s?

In this example, we're asked to find the rate of change of the radius, given the rate of change of the volume.

The formula that relates the volume and radius of a sphere to one another is simply the formula for the volume of a sphere.

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$$V = \frac{4}{3}\pi r^3$$

Before doing anything else, we use implicit differentiation to differentiate both sides with respect to time t.

$$\frac{dV}{dt} = \frac{4}{3}(3)\pi r^{3-1}\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

Now we plug in everything that we know. Keep in mind that dV/dt is the rate at which the volume is changing, dr/dt is the rate at which the radius is changing, and r is the length of the radius at a specific moment.

Our problem tells us that the rate of change of the volume is 400, and that the length of the radius at the specific moment we're interested in is 100.

$$400 = 4\pi(100)^2\frac{dr}{dt}$$

Solving for dr/dt gives

$$400 = 40{,}000\pi\frac{dr}{dt}$$

Therefore, we know that the radius of the balloon is increasing at a rate of $1/100\pi$ cm per second.

$$\frac{1}{100\pi} = \frac{dr}{dt}$$

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